EM for Poisson mixture

For Master 2 Math SV

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We are interested in the mixture of Poisson distributions.

Let us consider a mixture of 3 Poisson distributions of means $(5, 10, 20)$ and proportions chosen as you want.

1. Simulate a sample of size $n = 100$ from that distribution.
2. Plot the histogram.
3. Write a function to compute to marginal likelihood for any value of K
4. Write the EM function for any value of K
5. Propose a method to initialize the parameters μ and ω fir any K
6. Run the EM for $K = 3$ and check the likelihood
7. Run the EM for other values of K. Compute the criterion you know to choose K. Are you able to find the K you used to simulate?
8. Decrease the number of observations n. Do you still find K?
9. We modify the data distribution considering the Gamma Poisson

$$\lambda_i | Z_i = k \sim \Gamma(\rho\mu_k, \rho) \qquad Y_i | \lambda_i \sim Pois(\lambda_i)$$

Plot the histogram. Compute the means by cluster and the variance. Compare with the previous data. Adjust the model with the Poisson mixture model. What about the number of classes?